Are clock comparison methods useful for pulsar timing?

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https://arxiv.org/abs/2005.13631 + https://arxiv.org/abs/2011.01912

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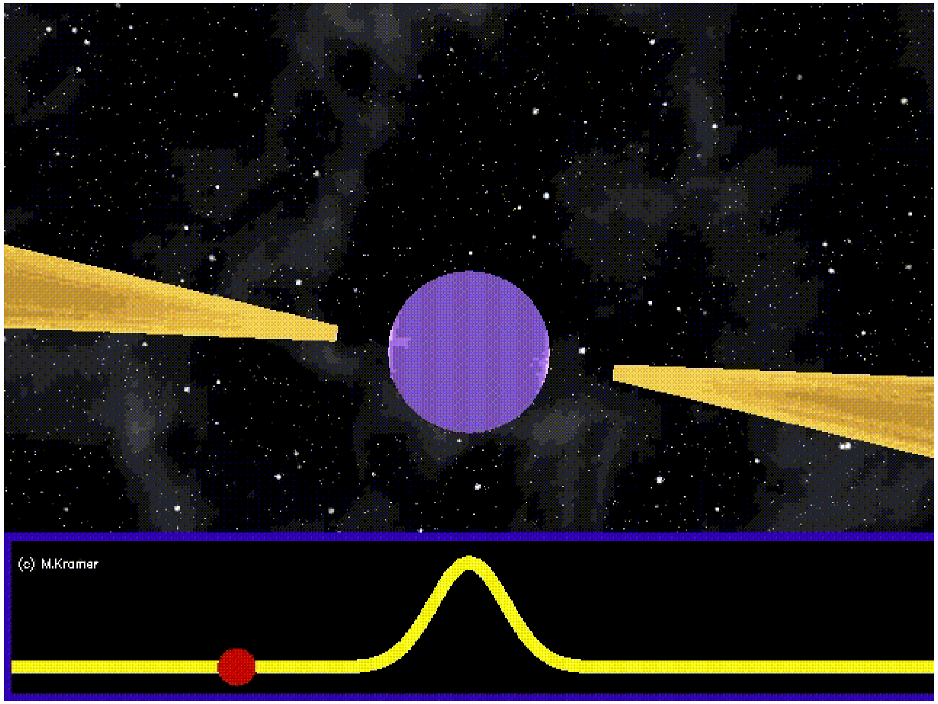
Overview

1) Introduction to Pulsar Timing
2) Spectra
3) Variances
4) Comparison with Pulsar Timing analysis
5) Summary

Pulsar Timing



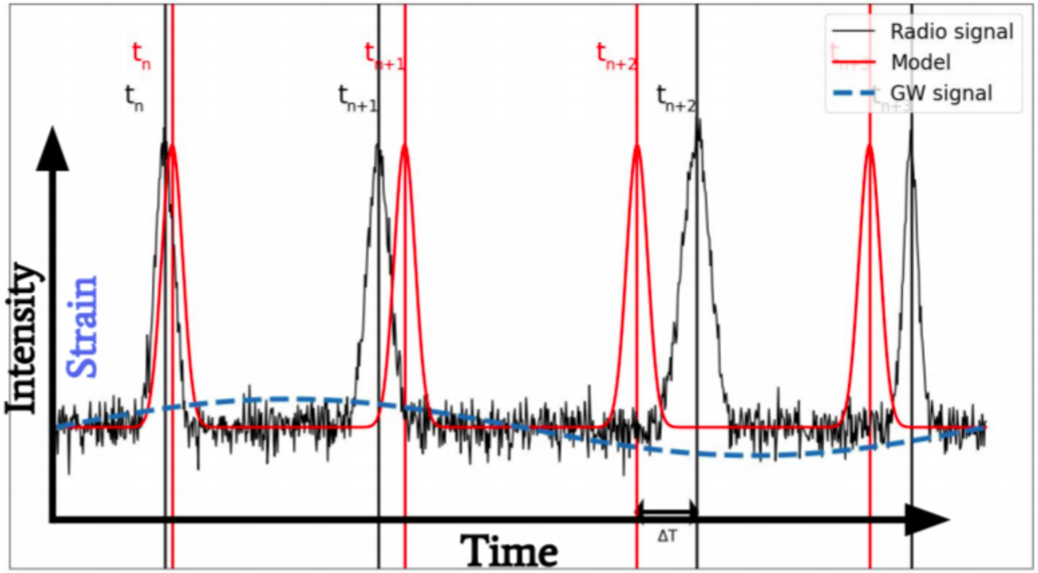
Credits: M Kramer

- Pulsars are spinning neutron stars
- Each rotation we get a radio pulse (lighthouse effect)
- Very stable astronomical clocks, especially millisecond pulsars
- Observations on a weekly-monthly cadence since 2-3 decades
- Pulsar timing model include parameters: sky position, proper motion, binary parameters, etc.
- Search for long-term variation in the residual time series, ie. red noise in low frequency
- Aim is to detect gravitational waves from an array of pulsars

Pulsar Timing

Credits: Nancay

Credits: M Falxa

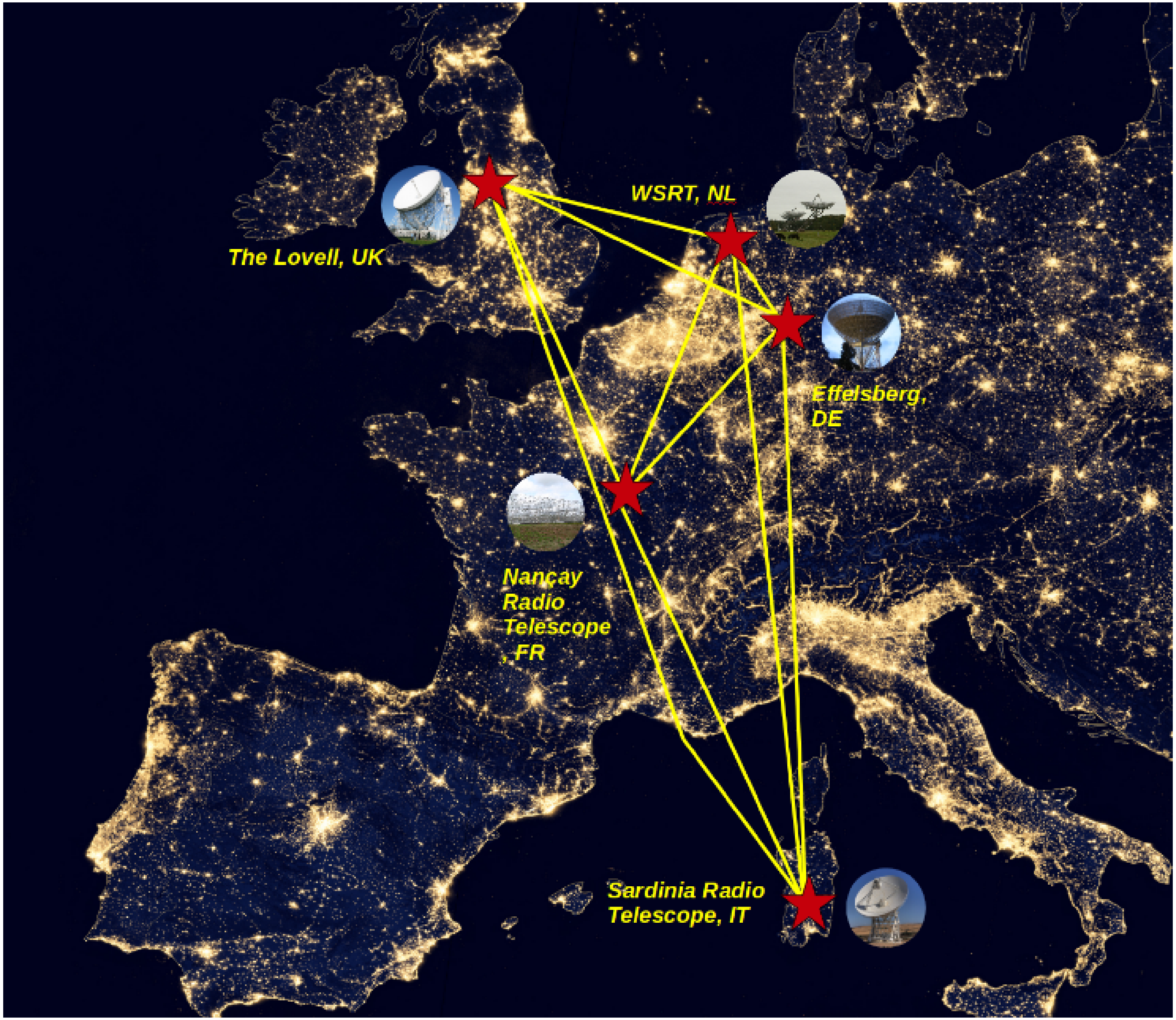


- Red noise: powerlaw with amplitude and spectral index
- White noise: E_k (EFAC) factor applied to the initial estimate W from the known telescope properties

$$S_{red}(f) = A_{PTA}^2 f^{-\gamma_{PTA}} \qquad S_{white} = (E_k W)^2 + Q_k^2$$

$$S(f) = S_{red}(f) + S_{white}$$

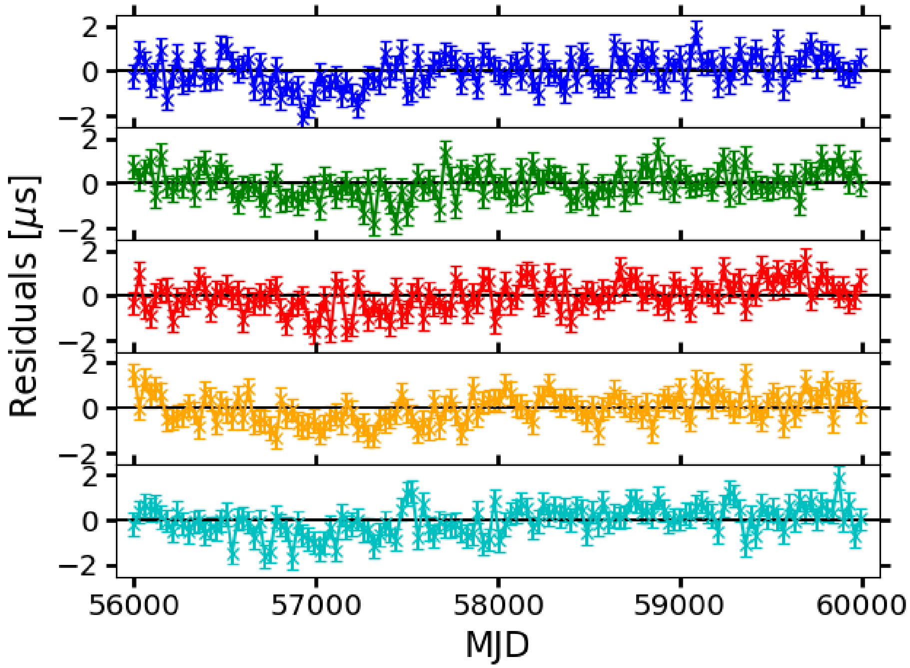
Large European Array for Pulsars (LEAP)



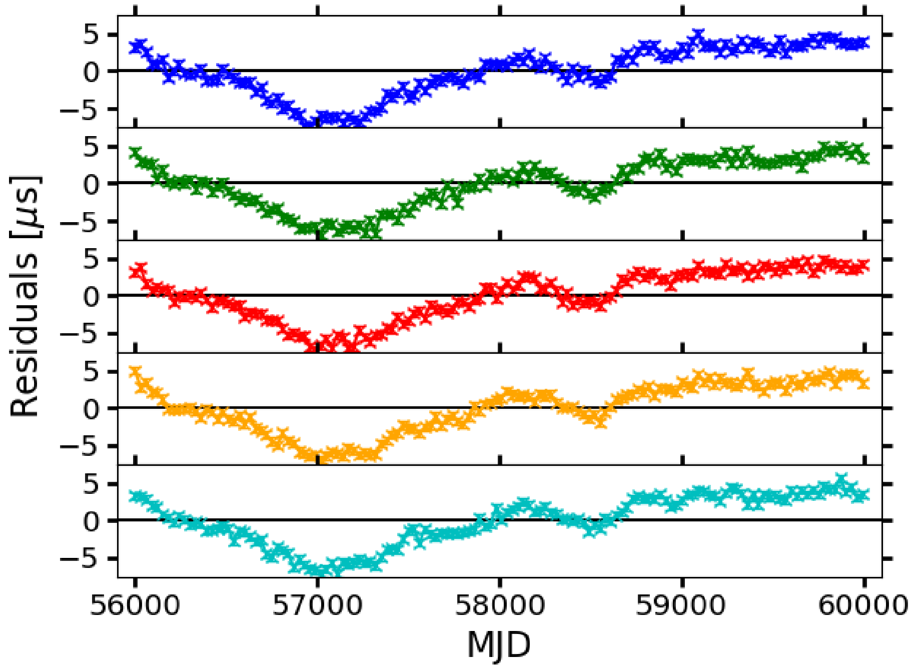
Credits: J McKee

Residuals

Boundary case



Red noise case



- Simulation of 5 simultaneous TOA series: ~10 years of observations, 30 days cadence, 0.5 us initial uncertainty, fix timing model
- Injection of different white noise realizations, but the same red noise realization
- Inspired by the observations of the Large European Array for Pulsars (LEAP) project

Bayesian methods – theorem

Prior: probability of the parameters

Likelihood: probability of producing the data given a set of parameters

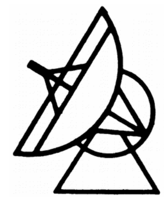
$$P(\theta|\mathscr{D}) = \frac{P(\theta)P(\mathscr{D}|\theta)}{P(\mathscr{D})}$$

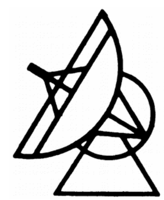
Posterior: probability of constraining the parameters given the data

Evidence: probability of the data

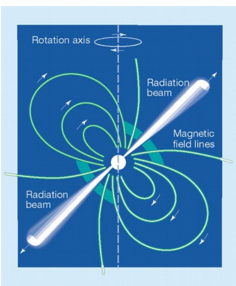
Normalization possible since: $\int P(\theta|\mathscr{D})d\theta = 1$

Spectral densities





X: signal c + noise(x)



signal c

Cross spectrum → signal c + reduced noise

Y: signal c + noise(y)

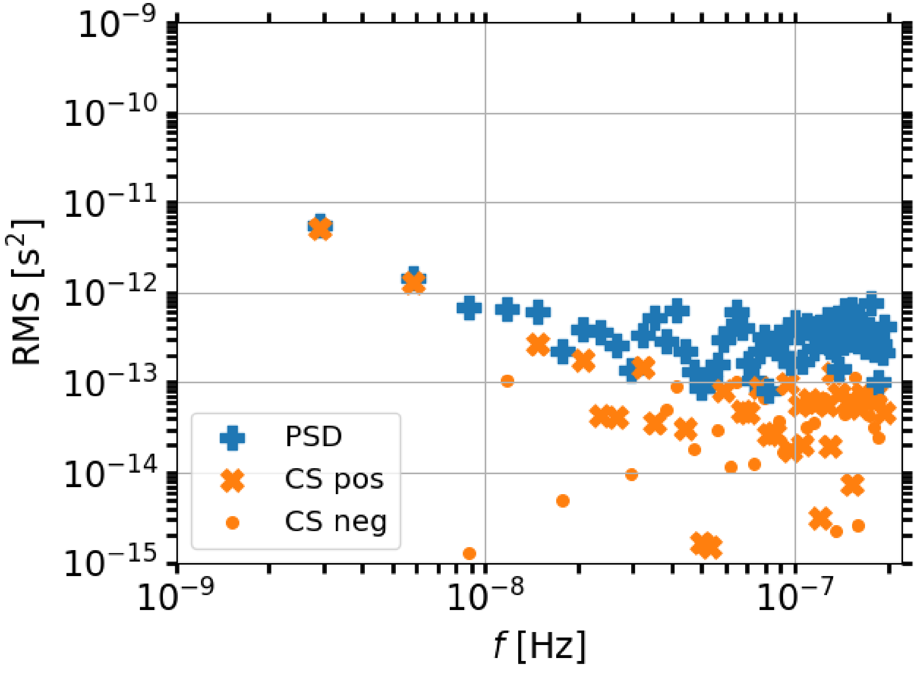
PSD: $$S_x(f) = \frac{2}{T_a} XX^*$$

CS: $$S_{yx} = \frac{2}{T_a} YX^* \quad \hat{S}_c = \frac{2}{T_a} \Re(S_{yx})$$

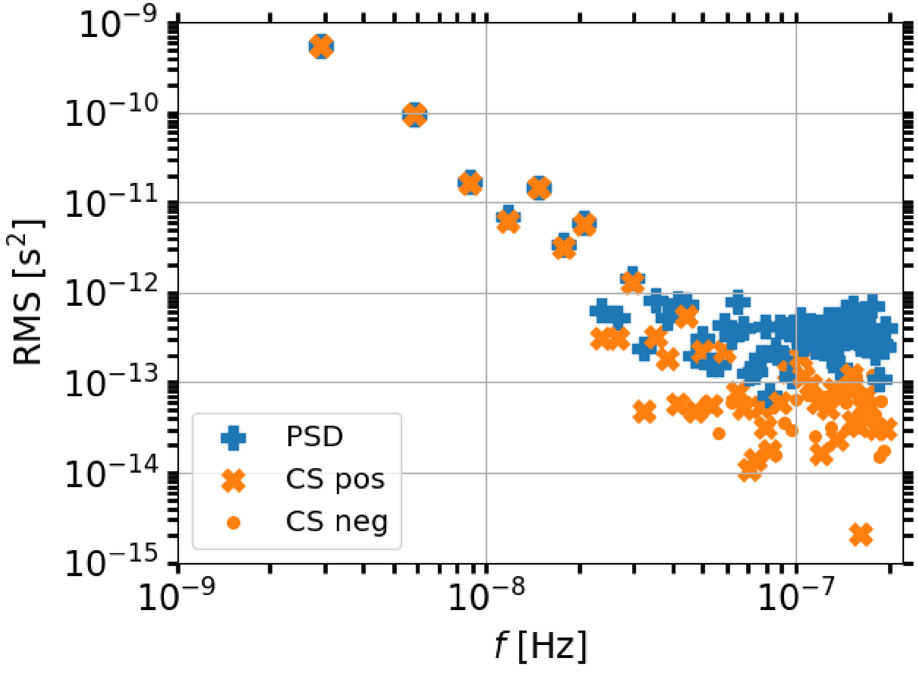
- Power spectral density (PSD) can be computed on each of the 5 residual series and then averaged → good white noise estimate
- Cross spectrum (CS) can be estimated on all combinations of 2 residual series (2 out of 5: 10 unique pairs) and then averaged → reduced uncommon noise

Fourier spectra

Boundary case

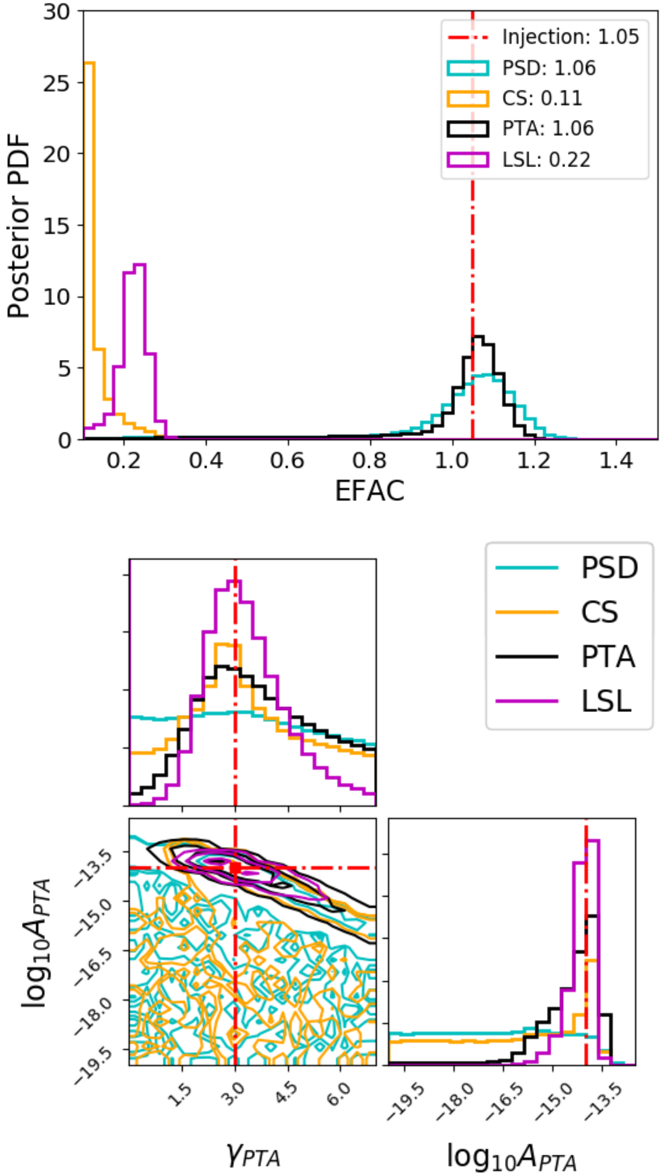


Red noise case

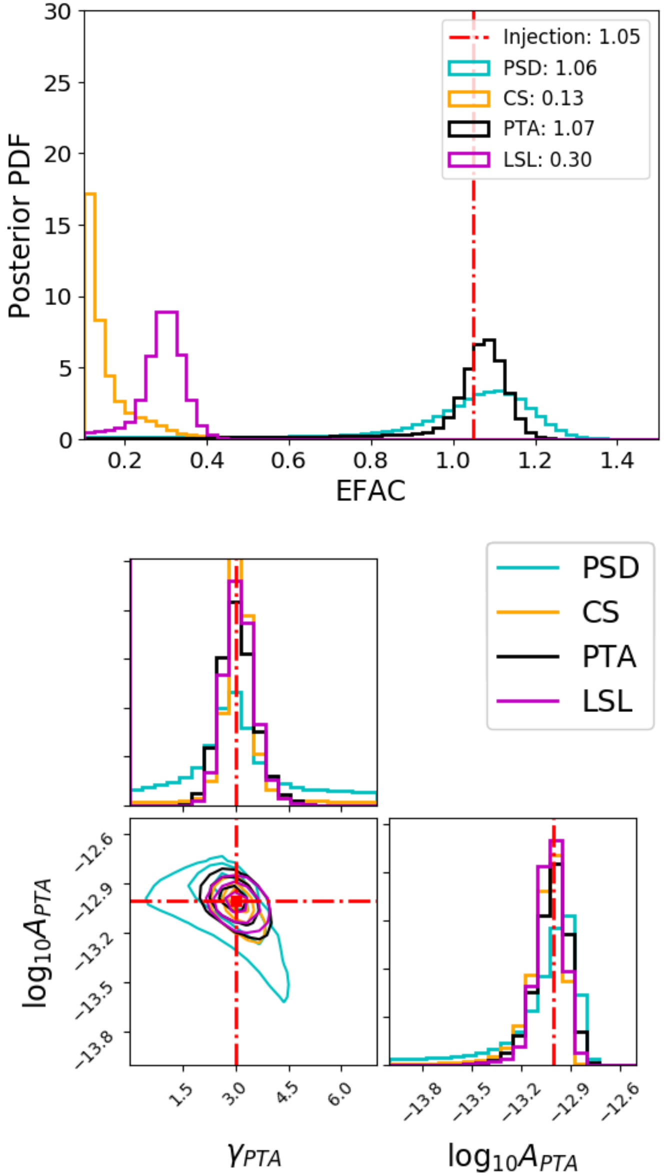


Spectral analysis – posterior distributions

Boundary case



Red noise case



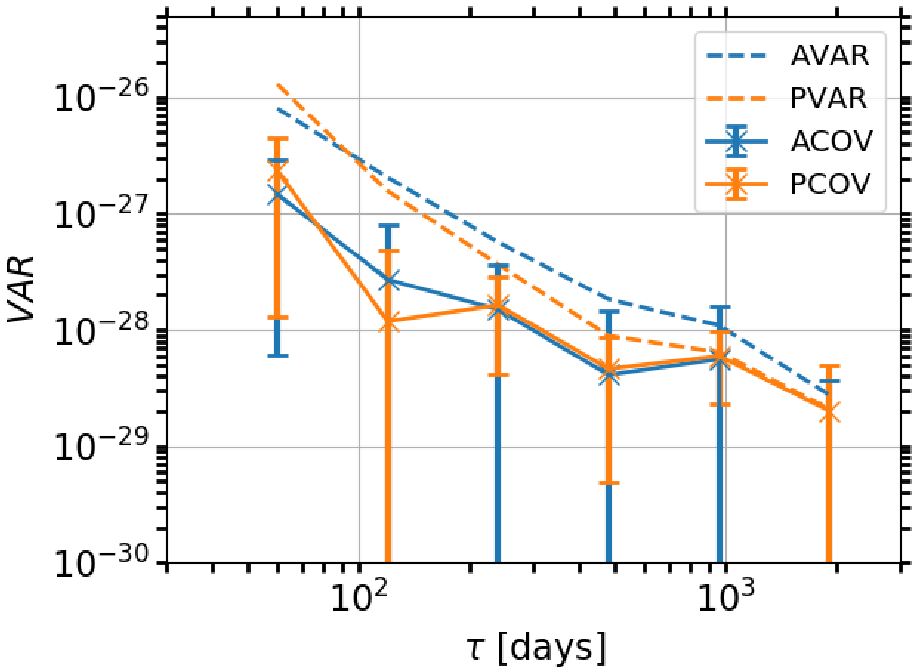
Two-sample variances

$$\mathrm{VAR} = \frac{1}{2}\mathbb{E}\left(\widehat{X_2} - \widehat{X_1}\right)$$

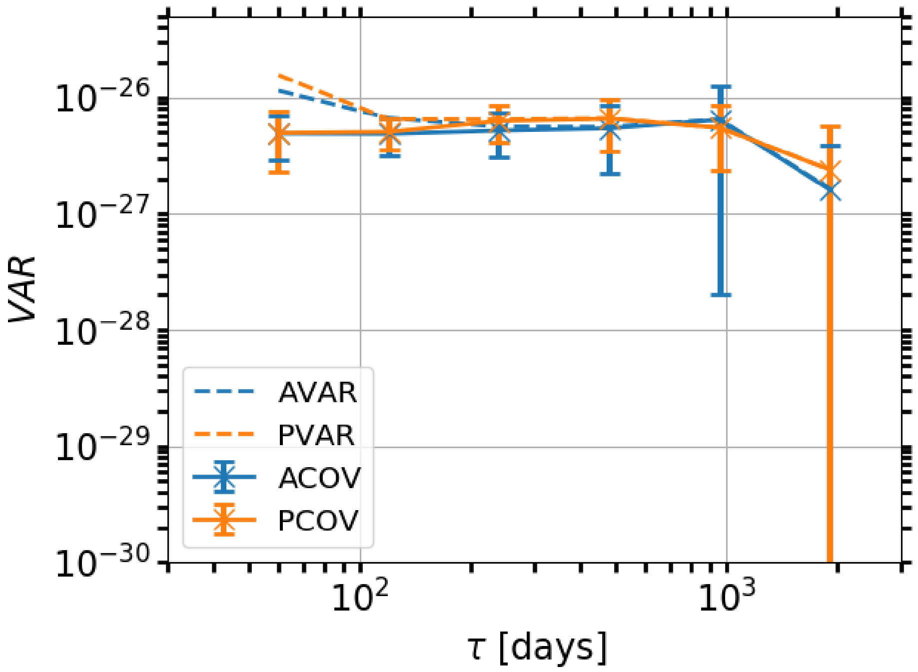
- We average the 5 residual series first, then compute the variance estimate
- Focus on Allan and Parabolic variances as two different estimators for X_2 and X_1
- Also use their covariances, where we average over the 10 different combinations of the 5 residual series

Variance examples

Boundary case

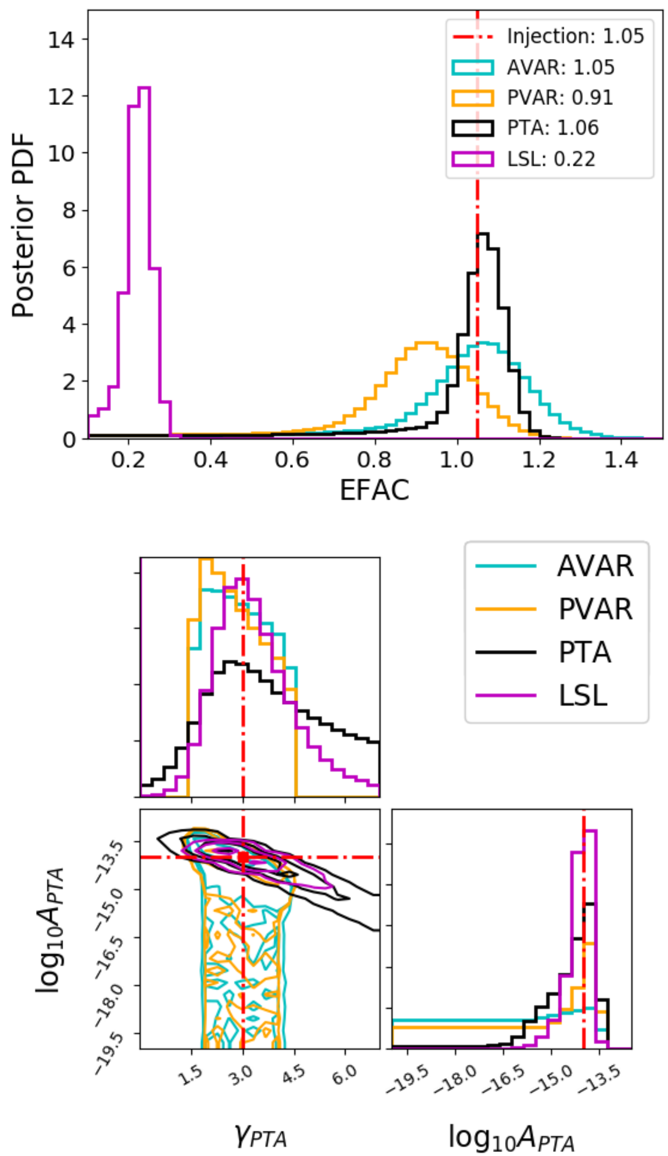


Red noise case

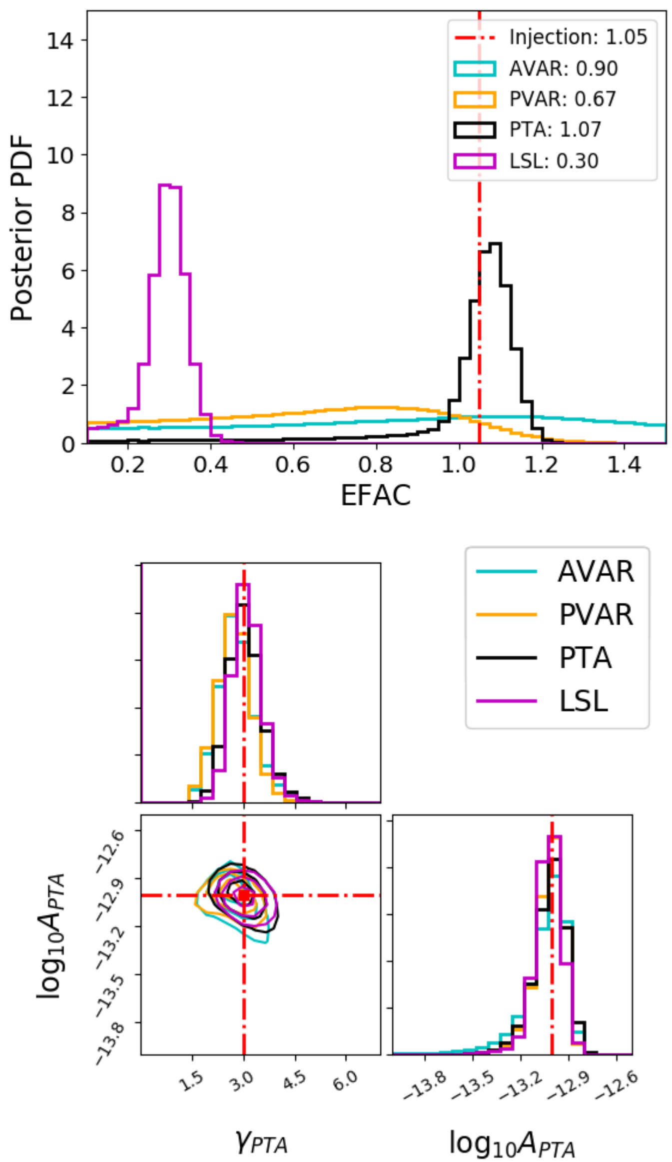


Variance analysis – posterior distributions

Boundary case



Red noise case



Summary

- Pulsar Timing Arrays (PTAs) have been timing pulsars very precisely for several decades
- PTAs look for long-term variation / red noise in the residual series
- Clock comparison methods can be applied to PTA data
- If the red noise is strong all methods will recover it well
- In a boundary case standard PTA methods tend to do better than the spectral methods and about the same as variances
- https://arxiv.org/abs/2005.13631
- https://arxiv.org/abs/2011.01912