Une problématique transverse :

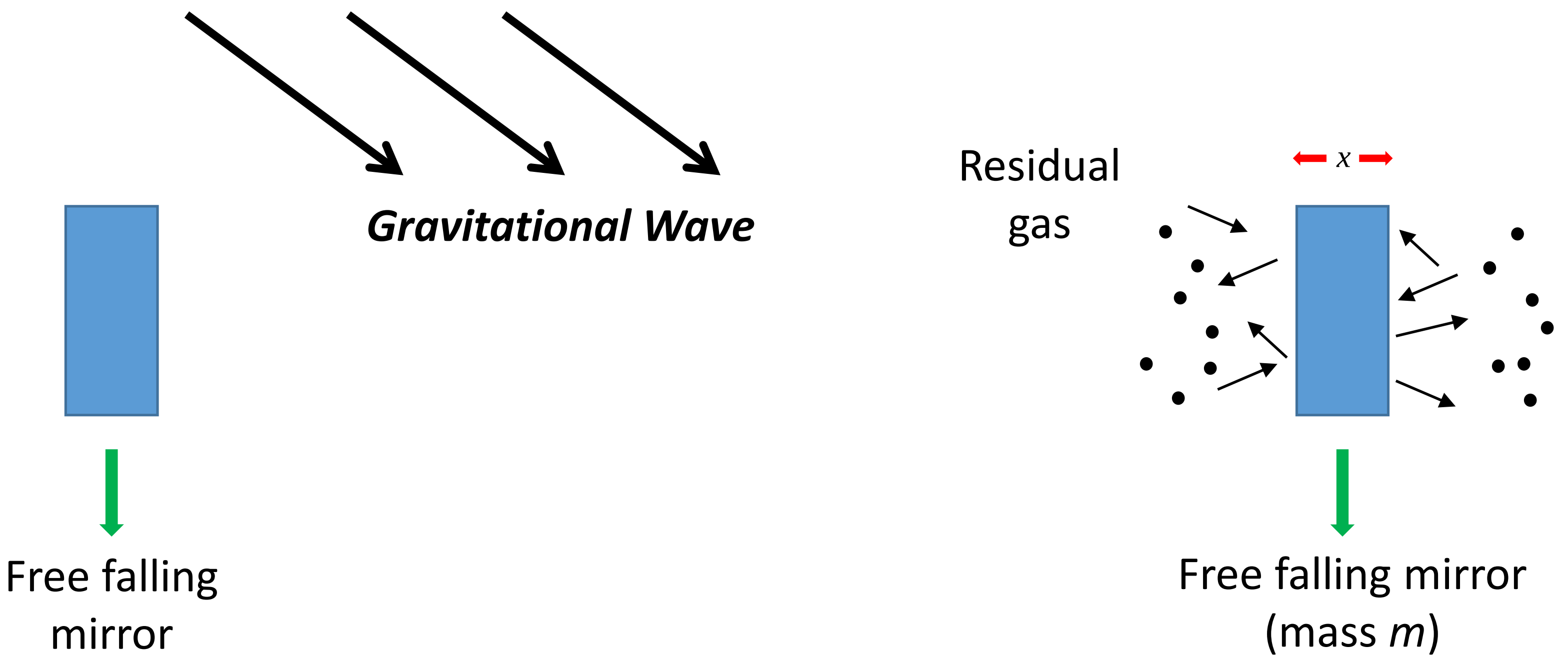
Le bruit thermique

Walid CHAIBI, ARTEMIS, Observatoire de la Côte d'Azur

Workshop PHOTNIQUE ET MESURES DE PRECISION

Mardi 11 octobre 2016

Thermal noise : a simple case on GW detector



<u>Langevin equation (1908)</u>

$$m\frac{dv_x}{dt} = -m\gamma v_x + F_L(t) \;,\; v_x = \frac{dx}{dt}$$

Viscous dissipation — Langevin force

Equipartition theorem ⇒

$$\gamma = \frac{\sigma_{F_L}^{\;2}}{4k_B T m}$$

⇒ "…relationship between equilibrium fluctuations and irreversibility…"

Callen and Welton, Phys. Rev. ***83*** *p34 (1951)*

Thermal noise : Fluctuation-dissipation therorem

$$S_{F_L}(f) = 4k_B T\, \Re(Z), \qquad Z = \frac{F}{v} : \text{Impedance}$$

Callen and Welton, Phys. Rev. ***83*** *p34 (1951)*

$$S_x(f) = \frac{S_{F_L}(f)}{|Z|^2 \omega^2} = \frac{4k_B T}{\omega^2} \times \frac{\Re(Z)}{|Z|^2} = \frac{4k_B T}{\omega^2} \times \Re\left(Z^{-1}\right) \longrightarrow \text{Thermal noise}$$

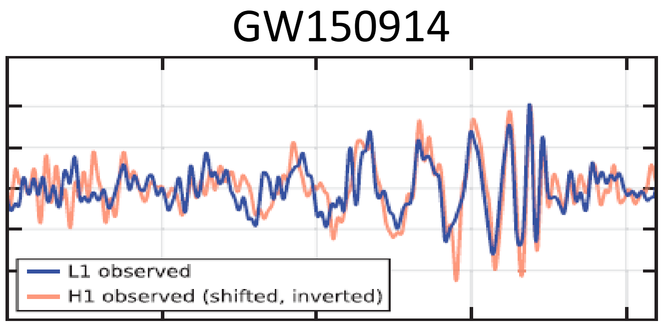
High mechanical *Q* factor → Low thermal noise

Y. Levin : energetic approach

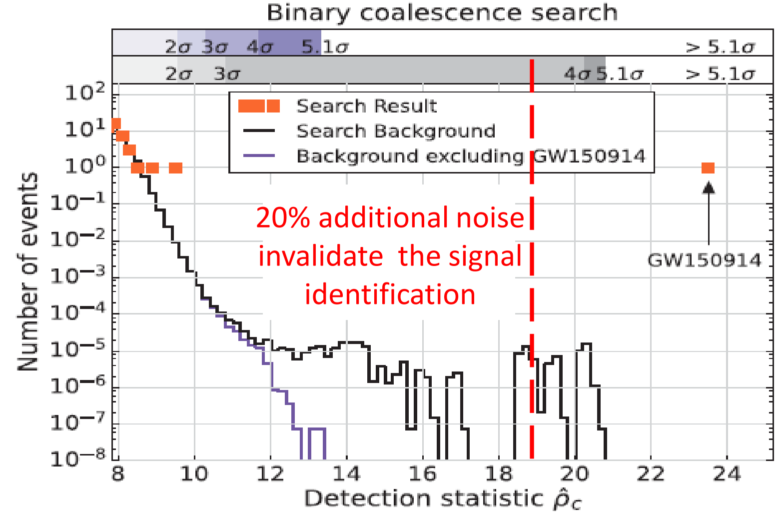
Y.Levin, Phys. Rev. D ***57*** *p659 (1998)*

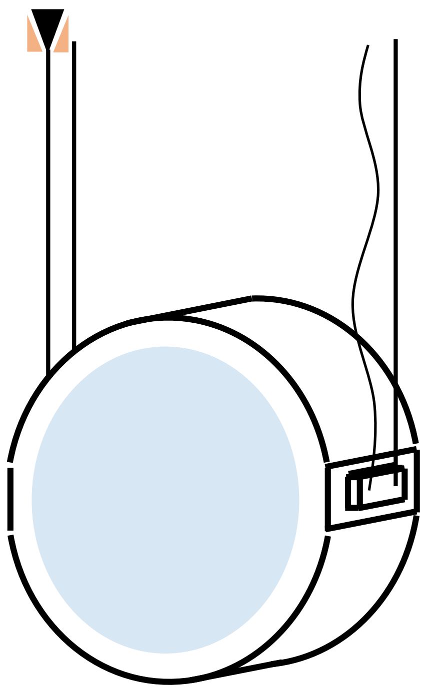
$$S_x(f) = \frac{8k_B T}{\omega^2} \times \frac{W_{diss}}{F_0^2}$$

Detection of gravitational waves



Phys. Rev. Lett. ***116*** *p061102 (2016)*





<u>Displacement noise</u>

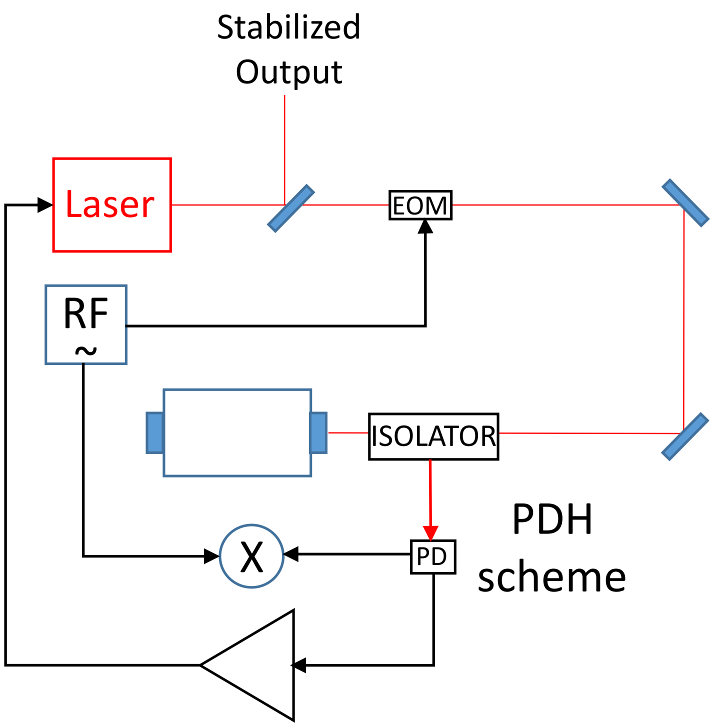
- Wires (pendulum + violin modes)
- Mirror's surface

<u>Mechanical losses</u>

- Mirror's internal losses
- Coating losses
- Magnetic control patches
- Ears silicate bonding
- Wires internal losses
- Surface losses
- Clamping losses

Thermal noise in optical frequency reference

High finesse reference cavity

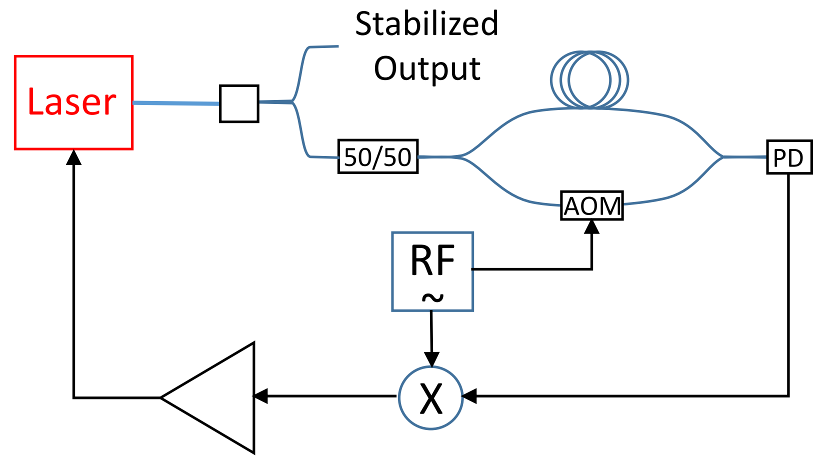


$$S_{out}(f) \simeq \frac{S_{\delta\nu}(f)}{f_p^2}$$

f_p : Cavity pole

Unbalanced fibered interferometer

Opt. Lett. ***34*** *p914 (2009)*

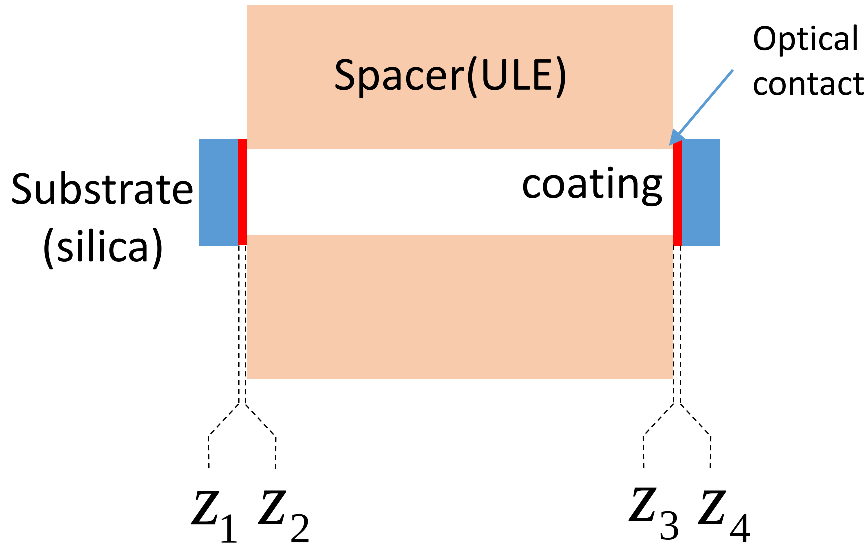


$$S_{out}(f) \simeq \frac{S_{\delta\nu}(f)}{f_0^2}$$

f_0^{-1} : Propagation time

$$\Rightarrow \quad \frac{S_{\delta\nu}}{\nu^2} = \frac{S_{\delta L}}{L^2}$$

High finesse cavities



$$L_{op} = (z_3 - z_2) + n_{eff}(z_2 - z_1) + n_{eff}(z_4 - z_2)$$

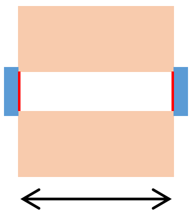
$$\left.\begin{aligned} \delta L_{op} = &\boxed{(\delta z_3 - \delta z_2)} \\ &+ n_{eff}(\delta z_2 - \delta z_1) + n_{eff}(\delta z_4 - \delta z_2) \end{aligned}\right\}$$

Thermo-mechanical effects

$$\left. + \delta n_{eff}(z_2 - z_1) + \delta n_{eff}(z_4 - z_2) \right\}$$

Thermo-refractif effects

High finesse cavities : different mechanisms



$\simeq 15\ \mathrm{cm}$

$$f \ll f_{resonance} < 10\ \mathrm{kHz}$$

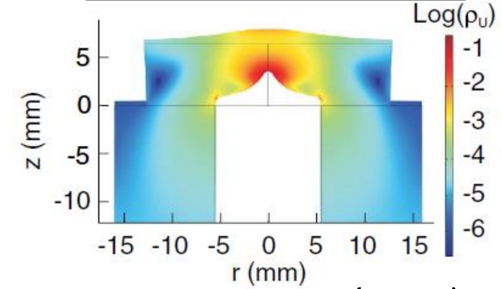
$$u_z(\omega) \simeq e^{i\phi(\omega)} \times u_{z\_stat} \simeq (1 + i\phi(\omega)) \times u_{z\_stat} \Rightarrow W_{diss} = \omega \times \phi \times U_{stat}$$

$$\phi(f_{res}) = Q^{-1}$$

$$S_x(f) = \frac{4k_B T}{\pi f} \sum U_{stat\_i} \times \phi_i$$

F_0 : Normalized to 1N

numerical simulations



JOSA B. ***29*** *p178 (2012)*

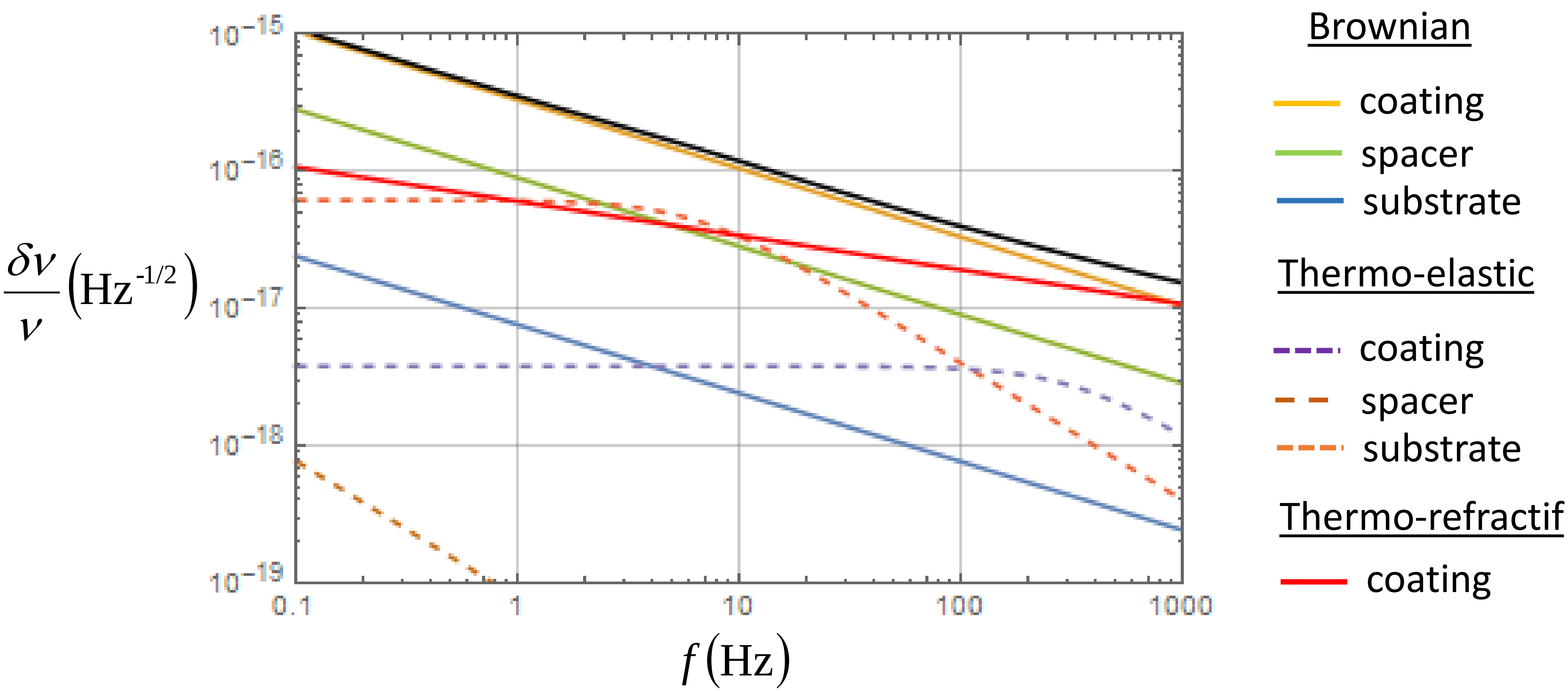
Dissipation mechanisms :

- Internal, surface : constant ϕ ⟶ Brownian noise ($\propto \sqrt{T}$)
- Thermo-elastic : due to thermal conductivity ⟶ Thermo-elastic noise ($\propto T$)

Statistical Physics : Landau & Lifschitz

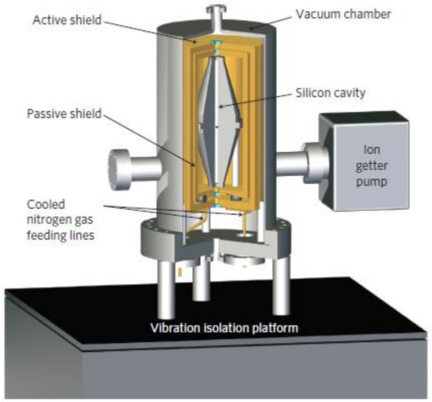
Related to local temperature fluctuations $\delta T(\vec{r})$ $\xrightarrow{\left(\frac{dn}{dT}\right)}$ Thermo-refractif noise

High finesse cavities : noise budget



High finesse cavities : thermal noise reduction

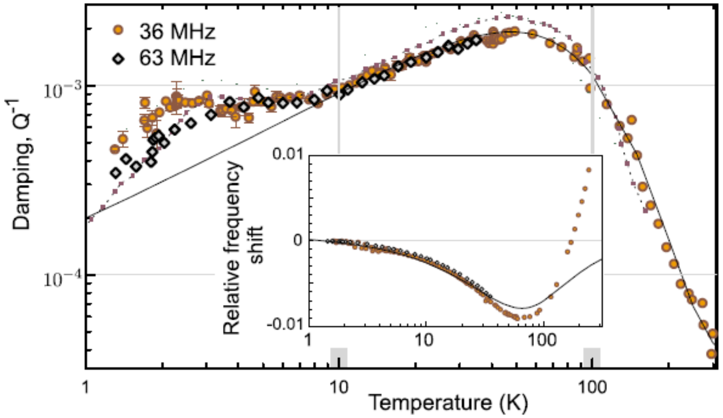
Cryogenic temperature



Silicon cavity @ 127 K : 10^{-16} at 1s

Nature Photonics ***6****, p.687(2012)*

Nano-oscillator Q factor

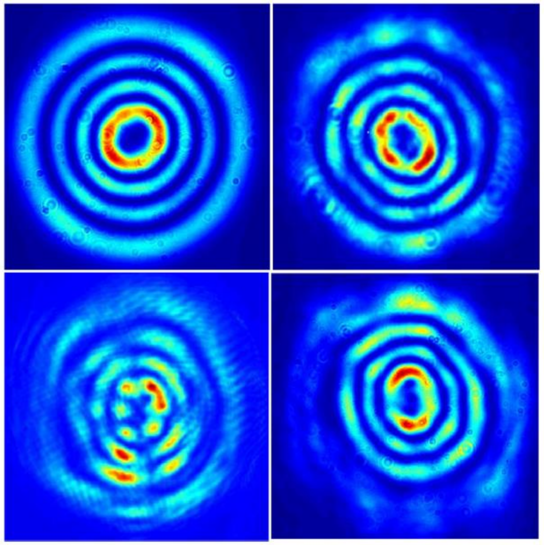


Phys. Rev. A ***80****, p.021803 (2009)*

Amorphous material..?

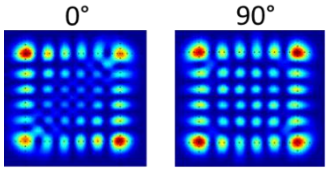
Large optical mode

LG modes into Glasgow 10 m prototype of GW detector

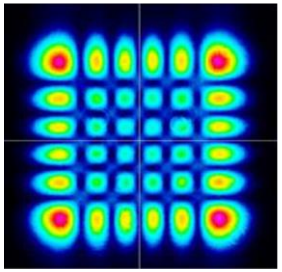


Class. Quantum Grav. ***30****, p.035004 (2013)*

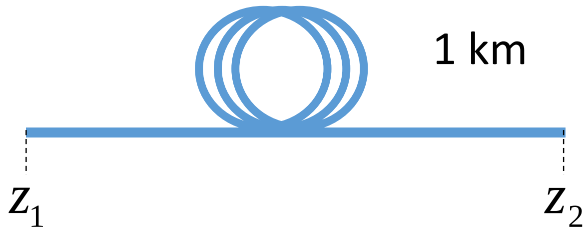
HG modes simulations



HG modes generation



Unbalanced fibered interferometer



$$L_{op} = (z_3 - z_2) + n_{eff}(z_2 - z_1) + n_{eff}(z_4 - z_2)$$

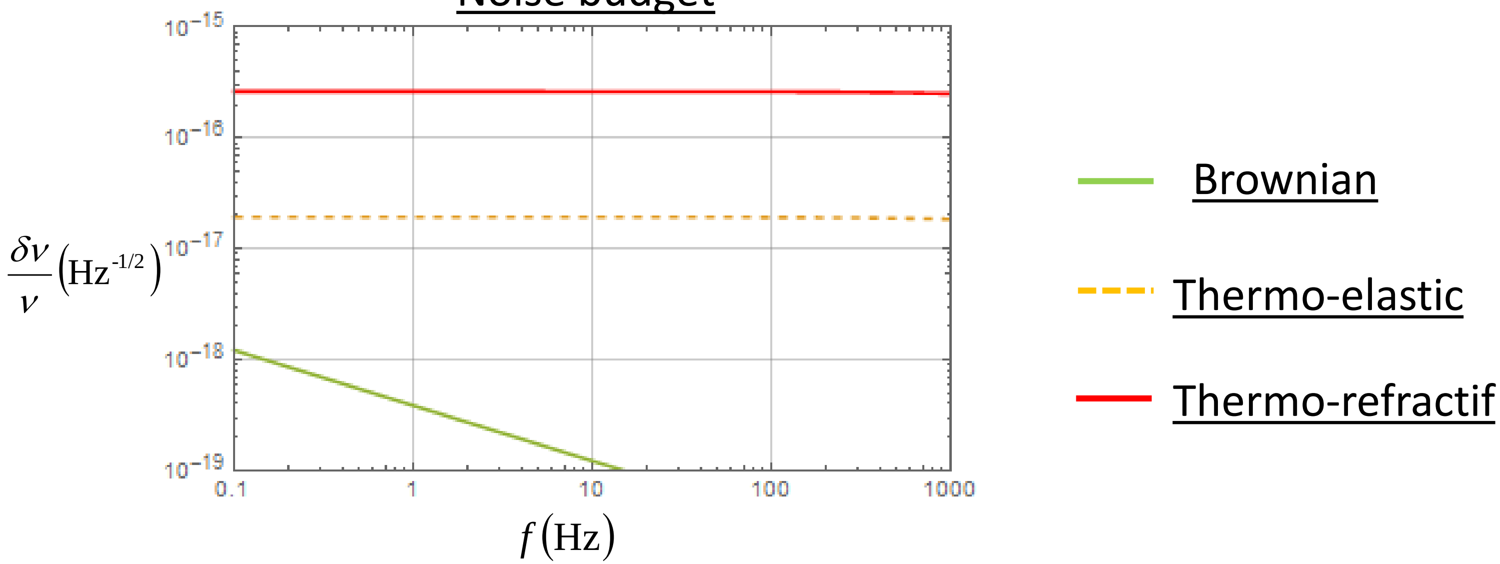
$$\delta L_{op} = \delta n (z_2 - z_1) \longrightarrow \text{Thermo-refractif}$$
$$+ n(\delta z_2 - \delta z_1) \longrightarrow \text{Thermo-elastic} \quad + \quad \text{Brownian}$$

*Opt.Quant. Elec. D **28** p43 (1996)*

$\Big\downarrow \langle \delta T(\vec{r}) \rangle$

$$\delta\nu / \nu \propto 1 / L^{1/2}$$

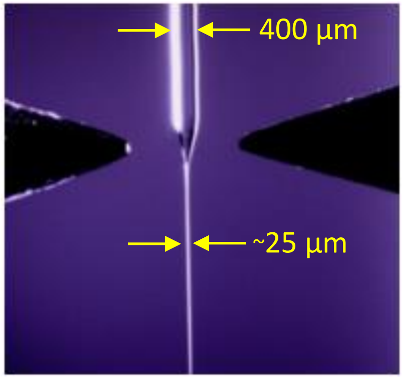
Noise budget

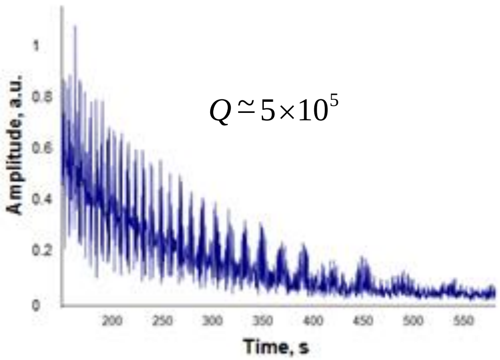


Conclusion : In practice…

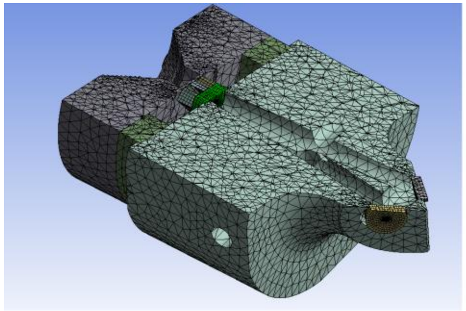
- Q factor measurement on isolated part (usually on a resonance)

Suspension for low frequency opto-mechanical oscillator





- Analytic calculations and/or Numerical simulations on the whole system



Pre-mode cleaner

- Hope for the best…